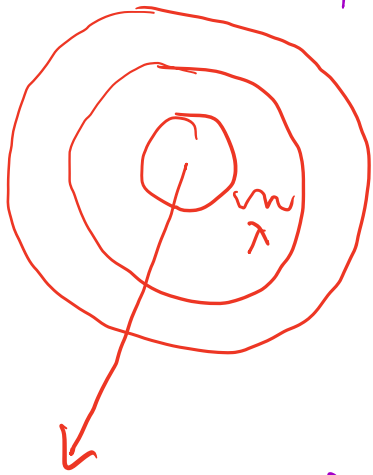


EM waves propagate in 3 dimensions



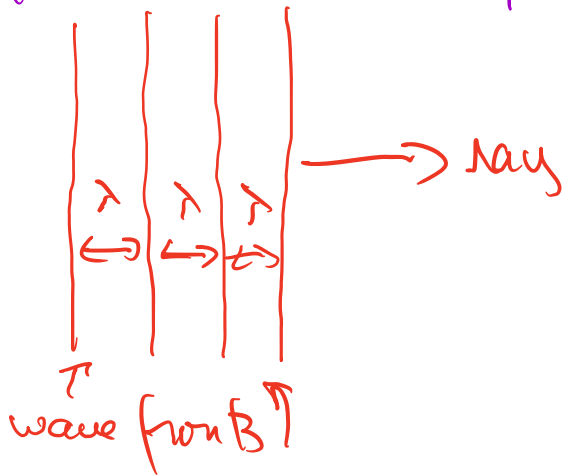
→ wave fronts
usually at some phase
dist between fronts
usually the wavelength

ray = direction of wave front at that point

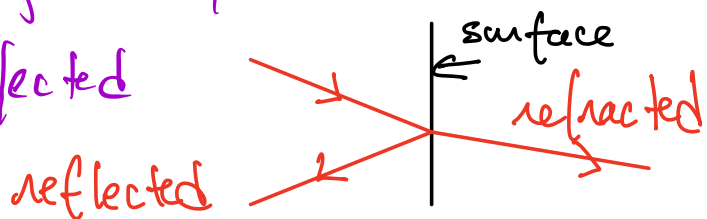
for EM radiation "infinitely far" from the source:

$$r \gg \lambda$$

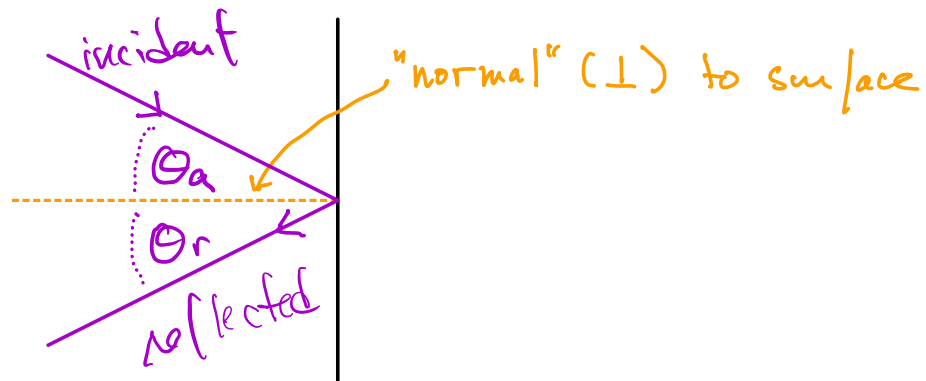
wave front \perp ray - plane waves



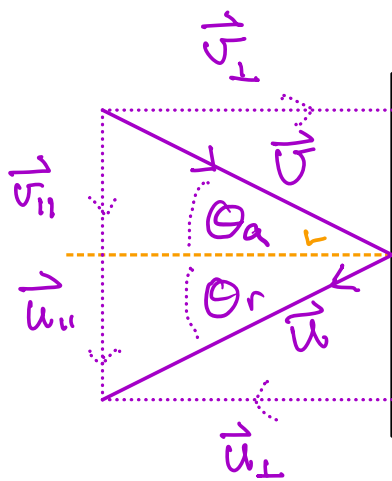
light waves hitting a surface can be both
transmitted and reflected



Reflection at a surface



specular reflection - from smooth surface like mirror



- surface has no unusual structure

\vec{v} = incident light

\vec{u} = reflected

parallel to surface

$$\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}$$

⊥ to surface

then $\tan \theta_a = v_{||} / v_{\perp}$

reflected light: $\vec{u}_{||} = \vec{v}_{||}$ so || unchanged
 $\vec{u}_{\perp} = -\vec{v}_{\perp}$ so ⊥ reverses dir

$$\vec{u} = \vec{u}_{||} + \vec{u}_{\perp} = \vec{v}_{||} - \vec{v}_{\perp}$$

reflected angle $\tan \theta_r = \frac{u_{||}}{u_{\perp}} = \frac{v_{||}}{v_{\perp}} = \tan \theta_a$

⇒ law of reflection: $\theta_r = \theta_i$

(not for diffuse reflection! - need very smooth surface)

Reflection at mirrors is almost perfect
Regular mirror, loss is few % loss (absorbed)
Can make highly reflective mirrors w/ special coating, loss < 0.001% or better

Refraction

speed of light is $c = 3 \times 10^8$ m/s in vacuum

in materials, light slows down

→ interaction w/ atoms & molecules

$$v_n = \frac{c}{n} \quad n = \text{index of refraction}$$

$n = 1$ for vacuum, $n > 1$ materials

$\sim \frac{3}{2}$ glass

$\sim \frac{4}{3}$ water

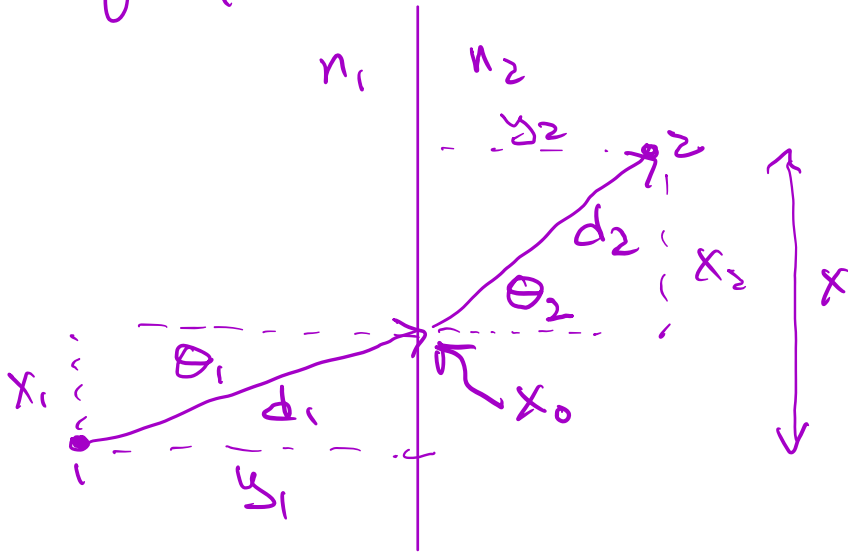
~ 2.5 diamond

note: in materials, freq f does not change!

$$c = \lambda f \quad \text{and} \quad v_n = \lambda_n f = \frac{c}{n} \lambda f$$

$$\therefore \Delta n = \frac{\lambda}{n} \quad \lambda \text{ is in vacuum}$$

law of refraction



principle of least time!

calculate time going from point 1 to point 2

$$t = t_1 + t_2 \quad t_1 = d_1/v_1 \quad t_2 = d_2/v_2$$

we can vary x_0 but not y_1 & y_2 and

not $x_1 + x_2$

$$d_1 = \sqrt{x_1^2 + y_1^2} = \sqrt{x_0^2 + y_1^2}$$

$$d_2 = \sqrt{x_2^2 + y_2^2} = \sqrt{(x - x_0)^2 + y_2^2}$$

$$\text{so } t = \frac{\sqrt{x_0^2 + y_1^2}}{v_1} + \frac{\sqrt{(x - x_0)^2 + y_2^2}}{v_2}$$

to find minimum take $\frac{dt}{dx_0} = 0$

$$\frac{dt}{dx_0} = \frac{1}{2} \frac{2x_0}{\sqrt{(1)^2 + x_0^2}} v_1 + \frac{1}{2} \frac{2(x-x_0)(-1)}{\sqrt{(2)^2 + (x-x_0)^2}} = 0$$

$$\text{or } \frac{x_0}{v_1 \sqrt{x_0^2 + y_1^2}} = \frac{(x-x_0)}{v_2 \sqrt{(x-x_0)^2 + y_2^2}}$$

$$\frac{x_0}{\sqrt{\quad}} = \sin \theta_1 \quad \text{and} \quad \frac{(x-x_0)}{\sqrt{\quad}} = \sin \theta_2$$

$$v_1 = \frac{c}{n_1} \quad \text{and} \quad v_2 = \frac{c}{n_2}$$

$$\text{so } n_1 \sin \theta_1 = n_2 \sin \theta_2$$

v = velocity in the medium

$n_1 = c/v_1$ for the medium.

for light $v = c$ in vacuum (and air)

in glass $v < c$ so $n = v/c > 1 \sim 1.5$

$$\text{or } v = c/1.5 \sim 2 \times 10^8 \text{ m/s}$$

Refraction: bending of light as it goes between 2 mediums

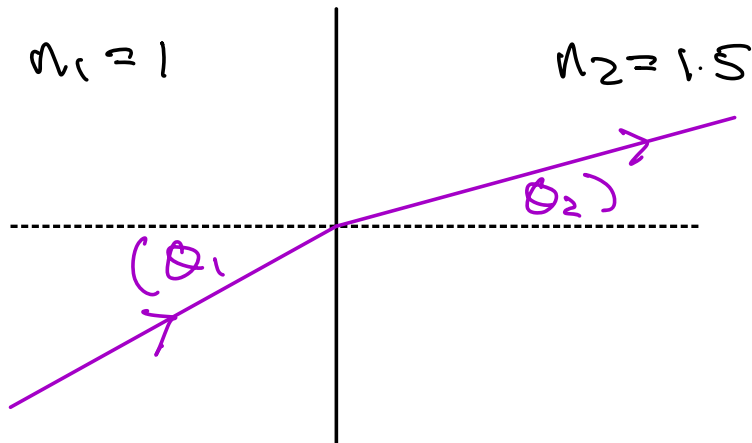
(air-glass, air-water, etc.)

Snell's Law: law of refraction

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

θ is always with respect to normal

cases: $n_1 < n_2 \Rightarrow \sin \theta_1 > \sin \theta_2$
 $\therefore \theta_1 > \theta_2$ so it bends towards normal



for $n_2 < n_1$, its opposite \rightarrow as if light goes backwards in time

note: there will also be some reflection at the boundary

for glass \sim loss is around 4% intensity

so $n_2 > n_1$ (from smaller to larger n)
 \Rightarrow bends towards normal

for $n_2 < n_1$ (from larger to smaller n)
 \Rightarrow bends away from normal

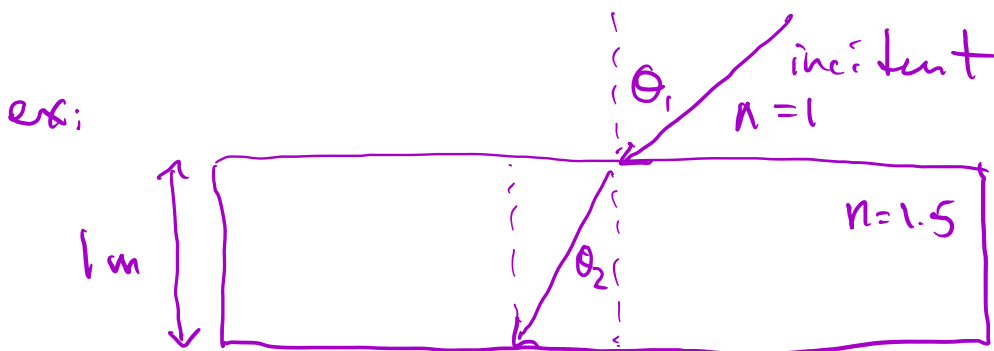
note: $c = \lambda f$ for light

f = vibration freq, λ = how far in the medium for 1 cycle

so if $v = \frac{c}{n} = \frac{\lambda f}{n}$ then $\lambda_n = \frac{\lambda}{n}$ wave length in medium

but f is the same

ex: light goes thru slab of glass



let $\theta_1 = 45^\circ$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_1 = \sin 45^\circ = \frac{1}{\sqrt{2}} = 1.5 \sin \theta_2$$

$$\theta_2 = \sin^{-1} \left(\frac{1}{1.5\sqrt{2}} \right) = \sin^{-1}(0.47) = 28.1^\circ$$

$$\tan \theta_2 = \frac{d}{\text{thickness}} = \frac{d}{1\text{m}}$$

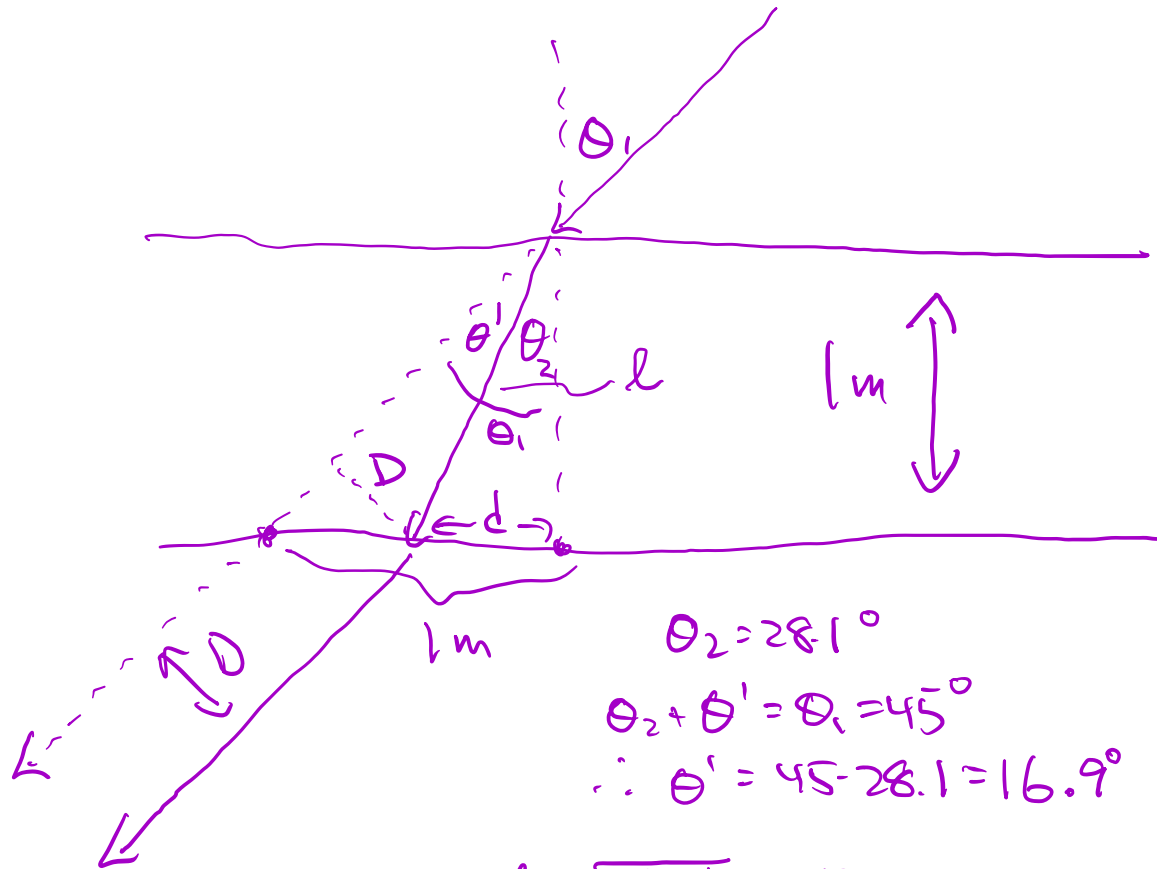
$$d = 1 \cdot \tan \theta_2 = 0.53\text{m}$$

what angle does light leave slab?

$n_2 \sin \theta_2 = n_1 \sin \theta_f \Rightarrow$ same eqn as when it entered

$$\text{so } \theta_f = 45^\circ$$

what is displacement D when it exits?

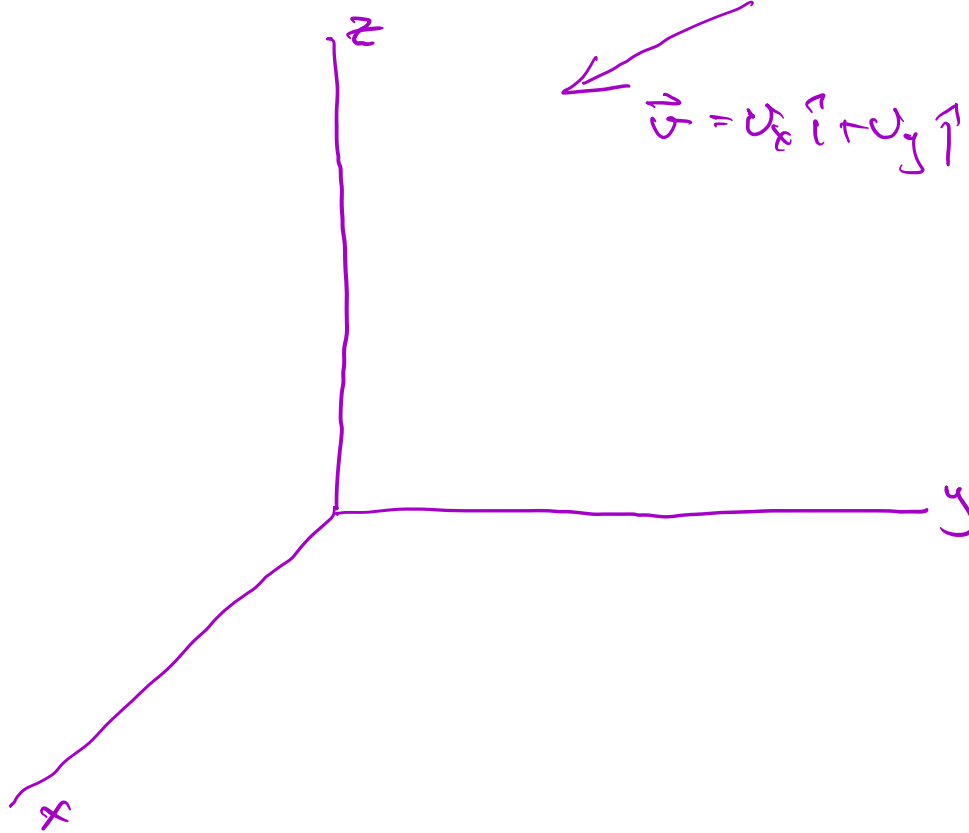


$$\theta_2 = 28.1^\circ$$
$$\theta_2 + \theta' = \theta_1 = 45^\circ$$
$$\therefore \theta' = 45 - 28.1 = 16.9^\circ$$

$$l = \sqrt{d^2 + 1^2} = 1.13 \text{ m}$$

$$D = l \sin \theta' = 1.13 \sin 16.9^\circ = 0.33 \text{ m}$$

Corner reflector



incident ray
 $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

1st reflection // xz plane - y component reverses

2nd " " xy " - z " "

3rd " " yz " - x " "

$$\vec{v}_f = -v_x \hat{i} - v_y \hat{j} - v_z \hat{k} = -\vec{v}!$$

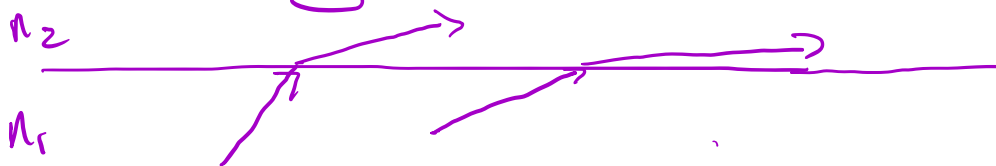
corner reflector will send beam back in same direction originated?

But slightly displaced

Total internal reflection

going from region 1 to 2 where $n_1 > n_2$

\Rightarrow bent away from normal



at some "critical angle" θ_c , $\theta_2 = 90^\circ$

$$n_1 \sin \theta_c = n_2 \sin 90^\circ = n_2$$

any angle $\theta_1 > \theta_c$ will not be transmitted out of region 1 \Rightarrow total internal reflection

ex: water $n = 1.5$, air = 1.0

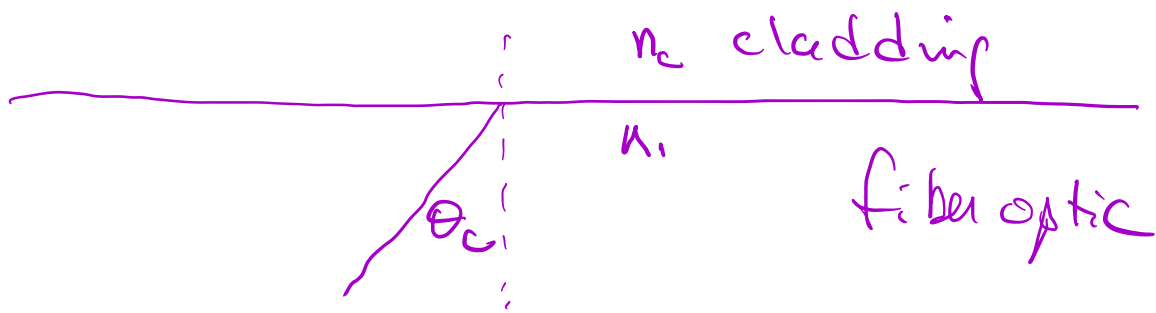
$$1.5 \sin \theta_c = 1 \Rightarrow \sin \theta_c = \frac{1}{1.5} \Rightarrow \theta_c = 41.8^\circ$$

remember θ_1 is angle wrt normal!

for water air $\theta_1 < \theta_c$ will be reflected

\Rightarrow what does a fish see from inside a aquarium?

\Rightarrow Discuss fiber optics & porro prisms for binocs



want to make critical angle small so that more light rays stay in fiber

$$\sin \theta_c = \frac{n_{\text{cladding}}}{n_{\text{core}}}$$

to minimize θ_c , minimize $\sin \theta_c \therefore$ make cladding small and core large

$n_{\text{cladding}} = 1$ smallest possible

glass: $n_{\text{core}} = 1.5$

$$\sin \theta_c = \frac{1}{1.5} = 0.67 \quad \theta_c = 41.8^\circ$$

diamond: $n_{\text{core}} = 2.4$

$$\sin \theta_c = \frac{1}{2.4} = 0.42 \quad \theta_c = 24.6^\circ$$

that means any light ray at $90 - 24.6 = 65.4^\circ$ wrt boundary will stay in!

Fiber optics use cladding & cores that have similar index of refraction

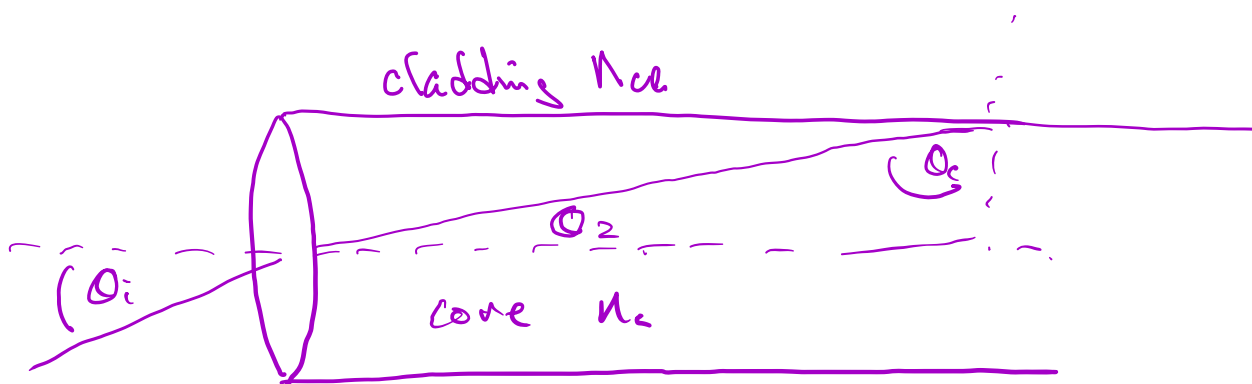
ex: core is silica doped w/ germanium $n=1.465$
cladding is pure silica $n=1.450$

$$\sin \theta_c = \frac{n_{\text{cladding}}}{n_{\text{core}}} = \frac{1.45}{1.465} = 0.99$$

$$\theta_c = 81.79$$



Aperture



θ_i is the entering angle in air, $n_i = 1$

$$\text{so } \sin \theta_i = n_c \sin \theta_2$$

$$\theta_2 + \theta_c = 90^\circ \text{ so } n_c \sin \theta_2 = n_c \sin(90^\circ - \theta_c) \\ = n_c \cos \theta_c$$

$$\sin \theta_i = n_c \cos \theta_c \Rightarrow \cos \theta_c = \frac{\sin \theta_i}{n_c}$$
$$\sin \theta_c = \frac{n_{cl}}{n_c}$$

$$\frac{\sin \theta_i}{n_c} = \cos \theta_c$$

$$\cos^2 \theta_c + \sin^2 \theta_c = 1 = \frac{\sin^2 \theta_i}{n_c^2} + \frac{n_{cl}^2}{n_c^2}$$

$$\sin^2 \theta_i = n_c^2 - n_{cl}^2$$

$$\text{or } \sin \theta_i = \sqrt{n_c^2 - n_{cl}^2}$$

for above case, $\sin \theta_i = 0.21$

$$\theta_i = 12.1^\circ$$

this is the "aperture" and tells you what entering angles will keep the light inside the fiber

\Rightarrow it is a very shallow angle to minimize light bouncing so that the pulse doesn't "spread"

Dispersion

"white light" is made up of all colors

some materials have index of refraction that is dependent on wavelength

⇒ this will disperse different colors during refraction

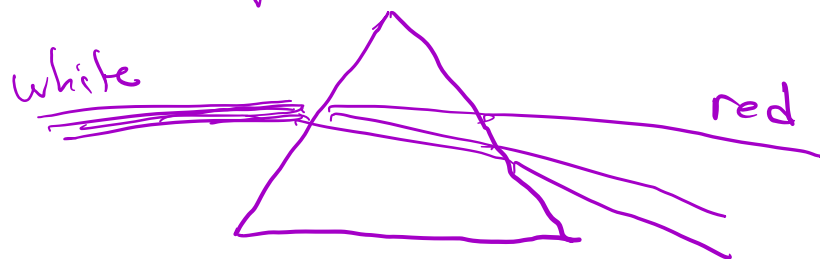
usually $n(\lambda)$ is bigger for smaller (blue)



This is very useful in separating wavelengths

Quartz, silicate flint glass, etc. is dispersive

Prisms disperse light



used in astronomy to detect ^{blue} wave lengths

Big bang: all galaxies are moving apart
=> length scales are increasing !

(balloon analogy)

=> the further away a galaxy is, the faster it's receding

Atomic spectra: atoms excite & deexcite
and emit photons
spectrum of light emitted identifies the
atom



astronomers use prisms to disperse light

so they can see the pattern

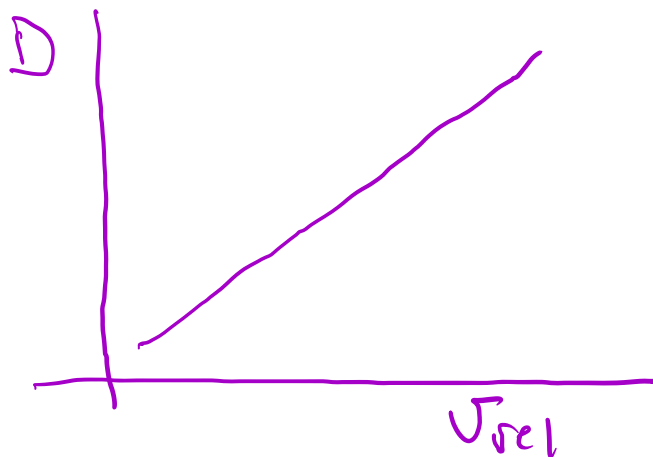
the wavelengths will be doppler shifted due to the relative motion

|| || || no velocity

|| || || relative velocity Δv
 $\Delta \lambda$ red shift related to v_{rel}

$$v = \lambda f$$
$$\Delta v = f \Delta \lambda \quad (\text{close enough, need special relativity})$$

Hubble's constant - relates distance to v_{rel}



by using prisms & atomic spectroscopy we can map the galaxies in the universe!

Polarization

this about the wave nature of light
(from wave equation $\Rightarrow \vec{E} = E_0 \hat{x} \cos(kx - \omega t)$)

here the vector nature is constant along \hat{x} direction

waves w/ constant direction are linearly polarized

can make materials that have "preferred" directions, will only let light thru if polarization is parallel to that direction

\Rightarrow called polarizing filter, or polarizer

Polarizer direction of EM wave is usually the direction of \vec{E} field, not \vec{B}

For polarizer filter, want direction to match to get transmission

Polarizer filters can pick out direction of light polarization

Unpolarized light

superposition of lots of light waves w/ random polarization



Polarizer filter will only allow light polarized along direction of polarization of filter to pass

Sunglasses filter intensity so light is not too bright

\Rightarrow if I_0 is initial intensity of unpolarized light, $I_t = I_0/2$
What happens when light hits a material?

\Rightarrow light = EM waves, w/ E fields
these fields accelerate atoms

Reflected light is the reemission away from the surface!

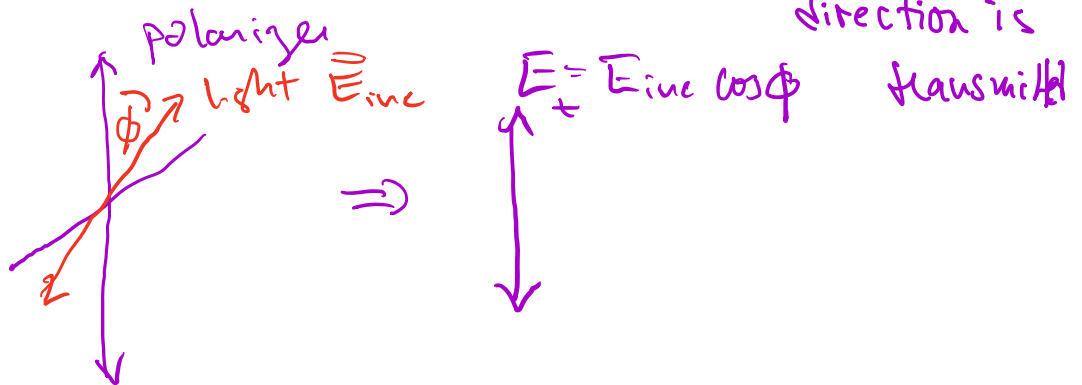
Refracted light is reemission inside the material

The treatment of reflection & refraction at the atomic level is very complex!

The thing to understand:

- light consists of oscillating E-fields that accelerate atoms
- accelerating atoms radiate light

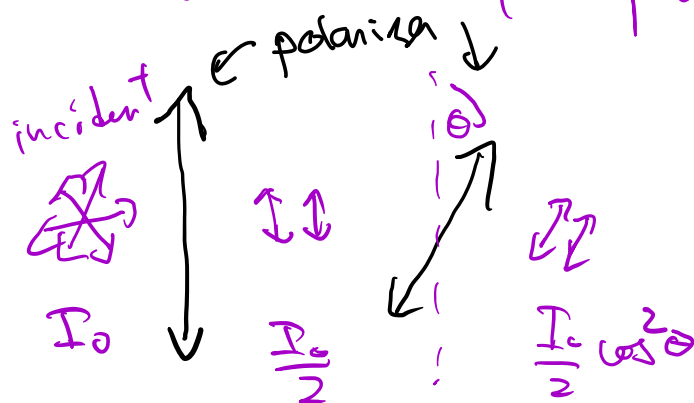
Polarized intensity - ^{only} light oscillating along polarizer direction is transmitted



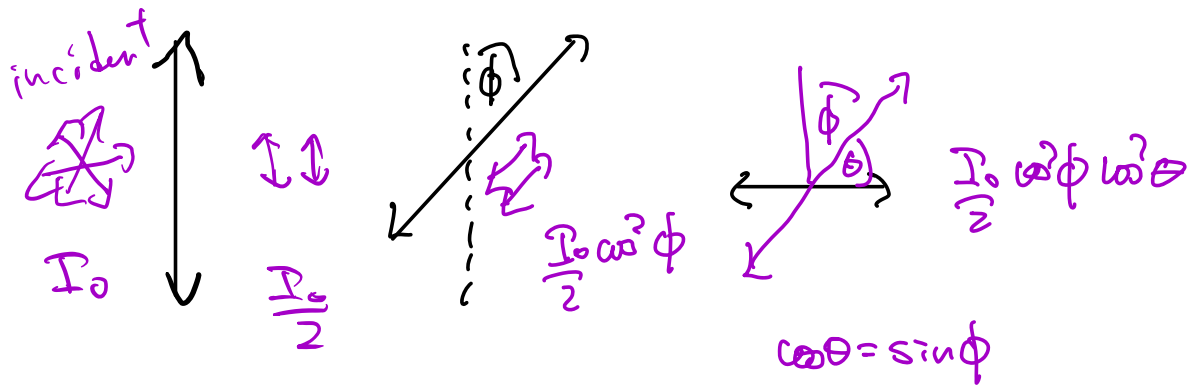
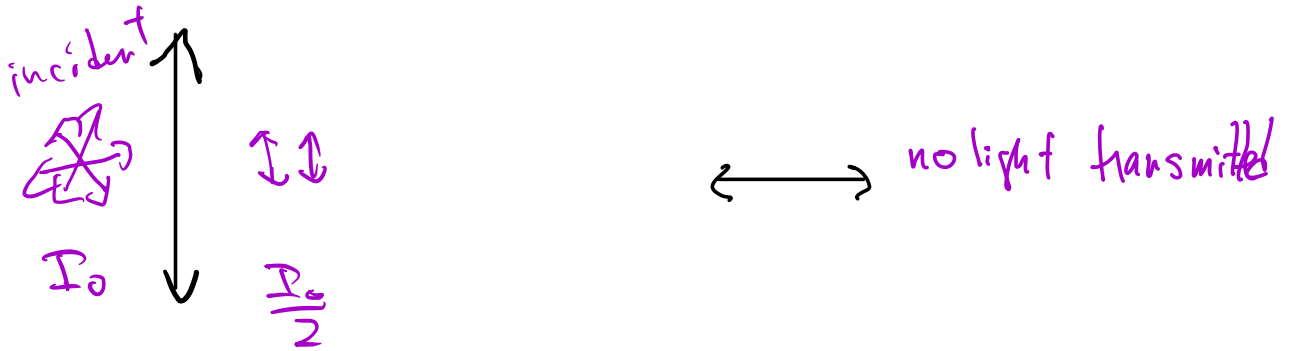
$$I_{inc} = \epsilon_0 E_{inc}^2 c$$

$$I_{trans} = \epsilon_0 E_{trans}^2 c = \epsilon_0 E_{inc}^2 \cos^2 \phi c = I_{inc} \cos^2 \phi$$

do demo w/ 3 polaroid filters

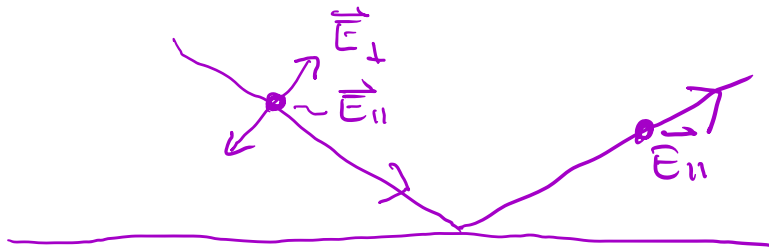


demo w/ 3 polarizers



$$\Rightarrow I_{\text{trans}} = \frac{I_0}{2} \cos^2 \phi \sin^2 \phi$$

Reflection at a surface



the incoming wave will have an oscillating E-field:

$$\vec{E} = \vec{E}_{||} + \vec{E}_{\perp}$$

$\vec{E}_{||}$ is parallel to surface

\vec{E}_{\perp} is \perp to surface

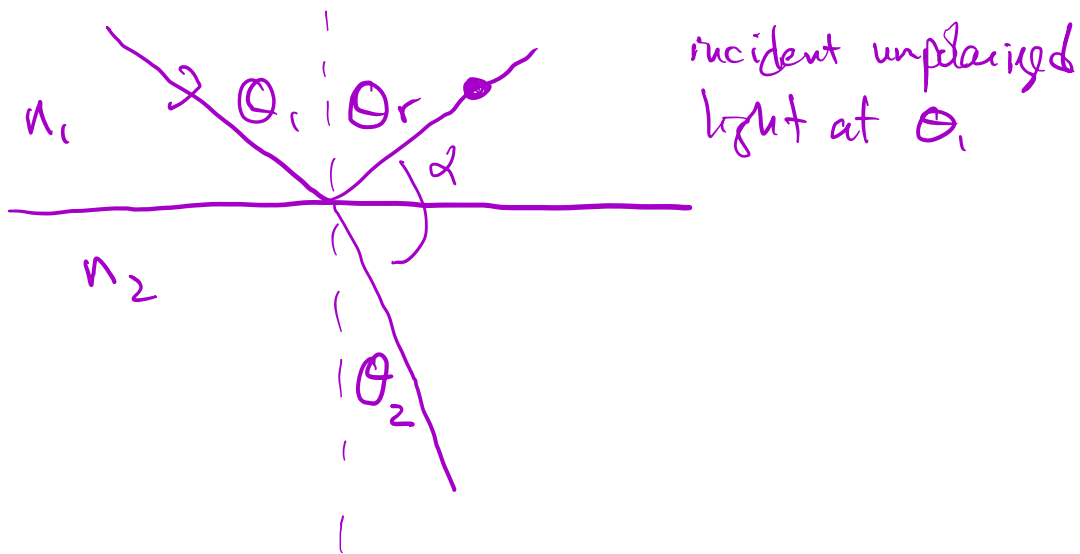
remember: $\vec{E} \perp$ direction of propagation

atoms on the surface will tend to move along surface and not into surface

so only $\vec{E}_{||}$ component causes acceleration

" " " " is reflected

\Rightarrow Reflected light is strongly polarized parallel to surface of reflection



reflected light is polarized parallel to ground
 refracted light has both \parallel & \perp polarizations

\Rightarrow what if $\theta_2 + \theta_r = 90^\circ$? then $\alpha = 90^\circ$

conservation of electric field: transmitted
 light is all \perp polarized so reflected
 light is completely polarized \parallel to surface

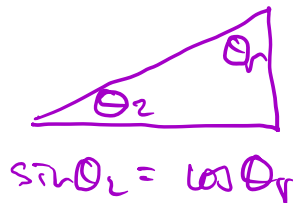
this condition:

$$\begin{aligned} n_1 \sin \theta_i &= n_2 \sin \theta_2 \\ &= n_2 \cos \theta_r \\ &= n_2 \cos \theta_i \end{aligned}$$

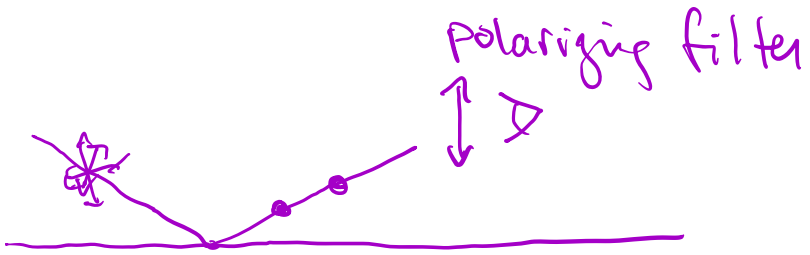
$$\theta_i = \theta_r$$

$$\theta_2 = 90 - \theta_r$$

so $\tan \theta_i = \frac{n_2}{n_1}$



polaroid sunglasses:

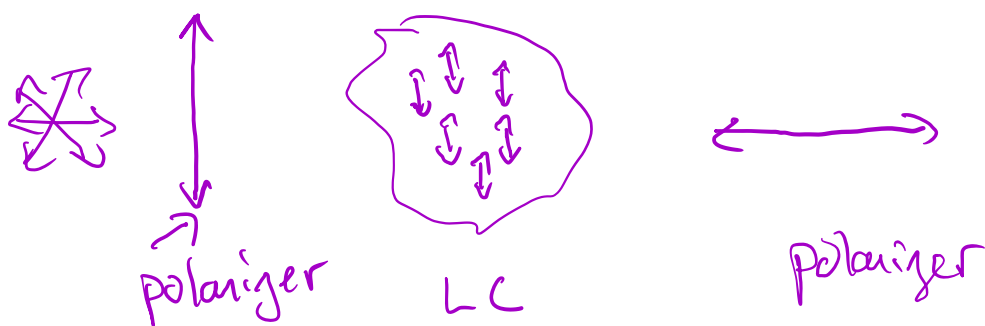


Liquid crystals

liquids w/ crystal domains like iron



- 1st polarizer will only allow 1 direction of polarization to pass
- LC will rotate light so it gets thru 2nd filter
- apply voltage to LC \therefore crystals line up



2nd filter blocks all light

voltages can control whether pixel is on/off

LCD: has array of pixels + back light
computer controls which pixel is on/off
color is implemented using color filters

⇒ contrast is important in LCD's

- can make LC layer thicker to increase contrast

- but switching LC direction is not instant

so thickness \times time \sim constant

faster LCD's mean less contrast

more contrast means can't refresh as fast

go over 2 polarizer + 3rd

do calculation

discuss LCDs